Child's Cognition of the Conservation of Continuous Quantities

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§1. Preface

Recently, it has been discussing actively whether we should expand the compulsory education to higher education or to infant education. And as this is a barometer which shows the security of people's living, such discussion will be welcome.

The author has been interested in the reports of investigation about number, quality and space which were done by the Geneva School, and the author has been reexaming the children in Fukushima prefecture as the object of his investigation, [1] so the discussion about infant education has been welcome in another sense mentioned above.

The process of child's cognition has its own proper process. So it is natural that the infant education will be doing according to the process. Especially learning according to the process is more important in the field of arithmetic education in order to grow up child's cognition of number and quantity.

The earlier methodical education of infant will be, the better it will be. In such meaning, the author has desired for the discussion about infant education.

This summer, the educational authorities stated needfullness of infant education, but it was too late. The exparts who take part in infant education have already set up *the conference of infant education* in spring 1962, and they are now practicing systematical education based on the cognition of children.

What process pursues the child's cognition of continuous quantities? This process is an urgent problem to establish the infant education systematically.

J. Piaget published the noticeable report based on his investigation.[2] According to his report,[3] constancy of continuous quantities is not gained at once, and the notion of conservation is gradually constructed. By grouping the answers to various questions, they are classified into three stages as following.

Stage Ⅰ. The child considers it natural for the quantity of liquid to vary according to the form and dimensions of the containers into which it is poured. That is, this is the stage of absence of conservation.

Stage Ⅱ. This is the stage which is a period of transition, conservation gradually emerges.

Stage Ⅲ. This is the stage which is the child at once postulates conservation of the quantities in each of the transformations to which they are subjected.

At first, we who participate in the education for children must pay attention to his theory.

J. Piaget used Swiss children as examinee for his investigation. The author uses children in Fukushima prefecture as examinee and is going to examine whether the stages which Piaget said are admitted or not. And if Piaget's results would be true, let's consider hereafter the best way which child experience in order to establish conservation of quantities.

§2. About investigations

(1) Date Feb. 18~21, 1963

(2) Examinee

Children in a nursery in Fukushima city

3 years old children	5 boys	3 girls	total	8 children
4 years old children	5 boys	5 girls	total	10 children
5 years old children	6 boys	5 girls	total	11 children
6 years old children	5 boys	5 girls	total	10 children

Children in a elementary school in Fukushima city

7 years old children	5 boys	5 girls	total	10 children
			sum-total	49 children

〔Notice〕 3 years old children are all member belonging to this nurcery, and the other children are selected at random.

(3) Co-workers

5 senior students of Educational Department of Fukushima University.

Hiroshi Asawa, Shigeo Suzuki, Takao Suzuki, Norio Tsugawa & Shuichi Naganuma.

The stuff of a certain nurcery and the stuff of a certain elementary school.

§3. The preparations and the technique of experiments

(1) The tools of experiment

① Many kinds of containers

A 2 300cc (Height 101mm Diameter 78mm)

B 4 100cc (Height 88mm Diameter 65mm)

C 8 50cc (Height 60mm Diameter 45mm)

L 2 250cc (Height 280mm Diameter 38mm)

② Juices

Red juice that dissolved red powder in water, Blue juice that dissolved blue powder in water

(2) The technigue of experiments

Investigators are five, one of them is operator who operates something to be necessary for questions and experiments, and the other two are assistance of operatar (they prepare for the next experiments and put away tools which have already used) , the last two record faithfully conversation and reaction between operator and examinee. And investigators pay attention sufficiently not to give shock for the examinee.

During the experiments we payed attention most carefully to produce atmosphere that children can speak without restraint.

We don't begin to investigate at once, but made effort to be familiar with children. For example, we talked with children; "Have you ever drunk blue juice ?", "What do you play every day ?" etc. Consequently investigation was made in very familiar atomosphere.

Besides, we said that we were recording conversation and reaction faithfully in shorthanded. This became useful later when we arrange the deta of investigation. Because we could recall the reaction of children in our minds, "Having the fidgets in his hand ……" or "Reply question seeing side ……" etc.

(i) The experiment of partition

In the experiment of partition (i) , the child is first given two containers of equal dimensions (A_1 and A_2) containing the same quantity of liquid (as is shown by the levels) . The contents of A_1 are then poured into two smaller containers of equall dimensions (B_1 and B_2) and the child is asked whether the quantity of liquid poured from A_1 into ($B_1 + B_2$) is still equal to that in A_2, etc. In this way, the liquids are subdivided in a variety of ways, and each time the problem of conservation is put in the form of a question as to equality or non-equality with one of the original containers.

We put red juice on the left side of examinee, and blue juice on the right.

(pic. 1)

red juice A_1 blue juice A_2

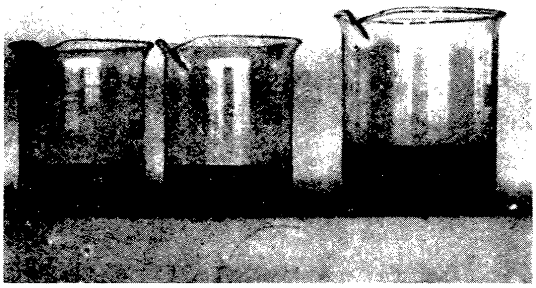
Height of juice 30mm

A_1 : red juice

A_2 : blue juice

Let's make the child recognize the contents are the same.

① (pic. 2)



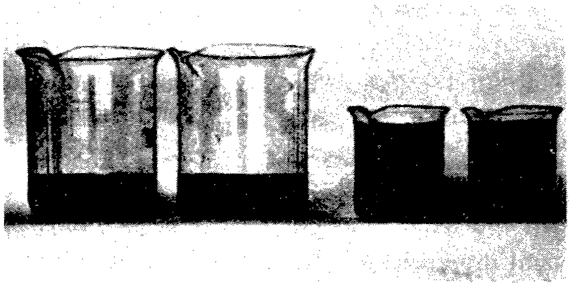
B_1 B_2 A_2

Height of juice in B_1, B_2 22mm

<Question>

Which do you think you can drink much more, A_2 or (B_1+B_2) ?

② (pic. 3)



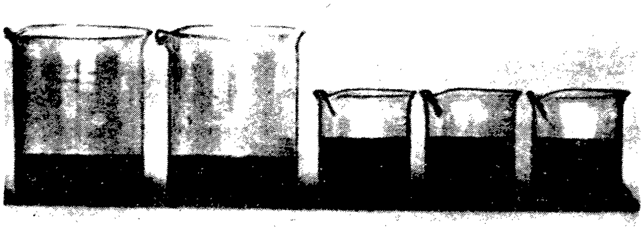
B_1 B_2 C_1 C_2

Height of blue juice in C_1, C_2 45mm

<Question>

Which do you think you can drink much more, (B_1+B_2) or (C_1+C_2) ?

③ (pic. 4)



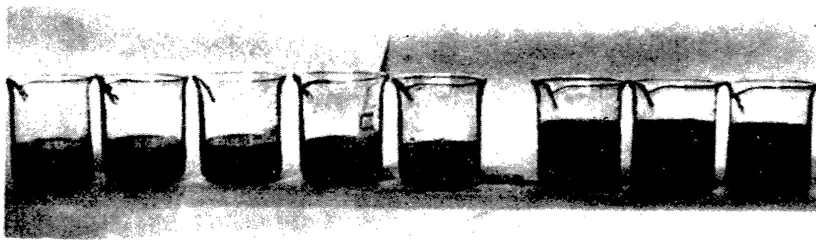
B_1 B_2 C_1 C_2 C_3

Height of blue juice in C_1 ~C_3 34mm

<Question>

Which do you think you can drink much more, (B_1+B_2) or $(C_1+C_2+C_3)$?

④ (pic. 5)



C_4 C_5 C_6 C_7 C_8 C_1 C_2 C_3

Height of red juice in $C_4 \sim C_8$ 24mm

<Question>

Which do you think you can drink much more,
$(C_4+C_5+C_6+C_7+C_8)$ or $(C_1+C_2+C_3)$?

⑤ $\boxed{C_1} + \boxed{C_2} + \boxed{C_3} \longrightarrow \boxed{A_2}$

<Question>

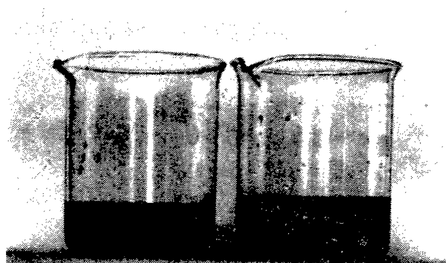
If you pour $(C_4+C_5+C_6+C_7+C_8)$ into A_1, where will the level come up? Please, point out.

(ii) The experiment of partition

In the experiment of partition (ii), the child is first given two containers of same dimensions (A_1 and A_2) containing different volume of liquid (as is shown by the levels). The contents of A_1 are then poured into two smaller containers of equal dimensions (B_1 and B_2) and the child is asked wether the quantity of liquid poured from A_1 into (B_1 + B_2) is equal to that in A_2 or not, etc. In this way, the liquids are subdivided in a variety of ways, and each time the problem of conservation is put in the form of a question as to inequality with A_2.

We put red juice on the left side of examinee, and blue juice on the right side.

(pic. 6)



A_1 : Height of red juice 30mm

A_2 : Height of blue juice 34mm

Let's make the child recognize the contents are $A_1 < A_2$.

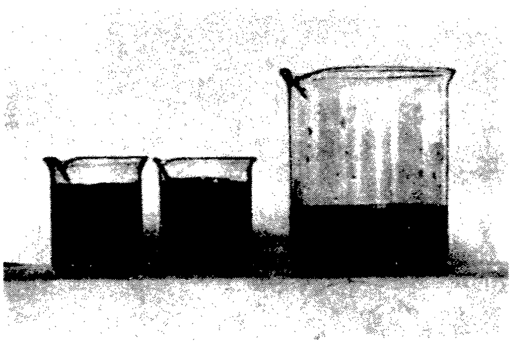
A_1 A_2

① $\boxed{A_1} \longrightarrow \boxed{B_1} + \boxed{B_2}$ Height of red juice in B_1 and B_2 22mm

<Quesetion>

Which do you thiuk you can drink much more, (B_1+B_2) or A_2 ?

② (pic. 7)



C_1 C_2 A_2

Height of red juice in C_1 and C_2 45mm

<Question>

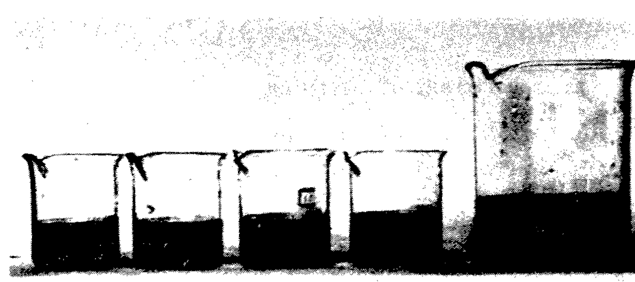
Which do you think you can drink much more, (C_1+C_2) or A_2 ?

③ (pic. 8)

Height of red juice in $C_1 \sim C_4$ 23mm

<Question>

Which do you think you can drink much more, $(C_1+C_2+C_3+C_4)$ or A_2 ?



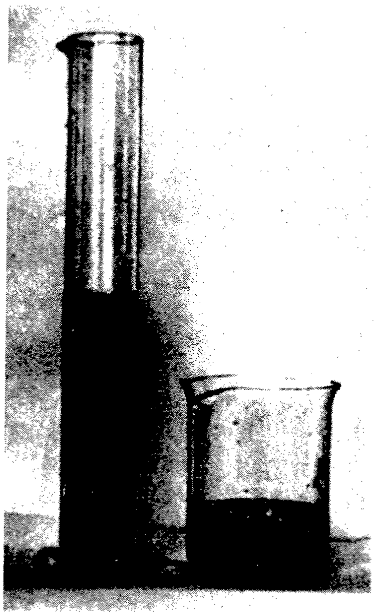
C_1 C_2 C_3 C_4 A_2

(iii) The experiment of multiplicative relations

In the experiment of multiplicative relations (iii) , the child is first given two containers of same dimensions (A_1 and A_2) containing the same quantity of liquid (as is shown by the levels) .

Then the child can be asked to pour the contents of A_1 into a container of a different shape L a quantity of liquid approximately the same as that in A_2.

(pic. 9)



L A$_2$

A_1 : Height of red juice 30mm

A_2 : Height of blue juice 30mm

Let's make the child recognize the contents are the same.

Pour as much red juice iuto this (L) as there is blue.

§4. The way of investigations and its results

We'll give representative examples for reference. And, I will classify these results in the stage I ～ III according to Piaget.

(i) The experiment of partition

○ Stage I

Yuzi (4 ; 8)

We put containers A_1, A_2 of red and blue juice (1/3 full) .

• "Are these red juice and blue juice the same amount ?"

"Yes."

• "Now, I will pour this red juice into these two other cups B_1 and B_2. (B_1 and B_2, which are thus 1/4 filled). Which do you think you can drink muck more, red juice or blue juice ?"

"It is red juice. "

• "Why ?"

"Because there are two."

We pour A_2 into C_1 and C_2, which are then almost full.

• "Now, which do you think you can drink much more ?"

"It is blue juice."

• "Why do you understand ?"

"Because, it is filled."

• "Let's devide these blue juice (C_1 and C_2) into three cups. Watch it carefully."

We pour a part of C_1 and C_2 into C_3. $C_1 \sim C_3$ are filled about 3/5.

- "Now, which do you think you can drink much more, red juice or blue juice?"
  "It is this (blue juice)."
- "Why ?"
  "Because, there are three."
- "Well, I will also pour this red juice into these little cups. Watch it carefully."
  We pour B_1 and B_2 into five cups $C_4 \sim C_8$. They are filled about 2/5.
- "Now, which do you think you can drink much more, this red juice or that blue juice ?"
  "It is this (red juice)."
- "Why ?"
  "Because, there are many cups in them."
  We pour back blue juice $C_1 \sim C_3$ into A_2.
- "I will pour this red juice into this large cup A_1, where will the red juice come up to ?"
  "……Here (he indicates the level as same height as A_2)."

As this child states the reason comparatively clearly, it is better example for investigation. But a great part of 3 years old children or 4 years old children can't state any reason ; for example "Because it is filled", or "I can't undestand." etc. And if we request answer he makes a temporary answer for the question. For example, if we ask him "Can you both the same amount to drink ?", he says "yes", but if we ask him again "Which do you think you can drink much more ?" he points out one of them, say "this juice or that juice". For this reason, we decided not to ask "Can you both the same amount to drink ?", but "Which do you think you can drink much more ?".

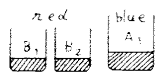
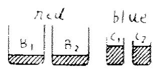
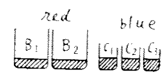
And if the child can't understand the significance of the question, we repeat the identical question many times until the child understands it.

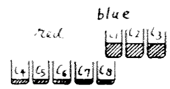
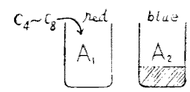
Moreover, when the child pointed to "this juice" for the question "Which do you think you can drink much more ?", we turned his attention to the other direction saying "*But this is high*" or "*But this is more filled*", then he was disturbed by the suggestion, and he changed his answer. There were many children who were similar to the above child.

Piaget says "For children at the first stage, the quantity of liquid increases or diminishes according to the size or number of the containers. The reasons given for this non-conservation vary from child to child, and from one moment to the next". [4]

Now, we get the same results as Piaget and this is obvious from (Table 1).

(Tab. 1) In what case children in stage Ⅰ, Ⅱ are bewildered by perception ?

											
stage	age \ answer		red is more	blue is more	same	red is more	blue is more	same	red is more	blue is more	same
Ⅰ	3	boy	3	1	0	0	1	2	2	1	1
		girl	2	0	0	2	1	0	2	0	0
	4	boy	2	3	0	1	4	0	0	5	0
		girl	2	3	0	1	4	0	4	1	0
	5	boy	1	4	0	0	4	0	1	4	0
		girl	2	3	0	0	5	0	3	2	0
	6	boy	1	3	1	0	5	0	0	5	0
		girl	2	2	0	0	4	0	2	2	0
	7	boy	0	1	0	1	0	0	0	1	0
		girl	2	0	0	2	0	0	1	1	0
	total	boy	7	12	1	2	14	2	3	16	1
		girl	10	8	0	5	14	0	12	6	0
		total	17	20	1	7	28	2	15	22	1
Ⅱ	3	boy	0	0	0	0	0	0	0	0	0
		girl	0	0	0	0	0	0	0	0	0
	4	boy	0	0	0	0	0	0	0	0	0
		girl	0	0	0	0	0	0	0	0	0
	5	boy	0	0	0	0	0	0	0	0	0
		girl	0	0	0	0	0	0	0	0	0
	6	boy	0	0	0	0	0	0	0	0	0
		girl	0	0	1	0	1	0	1	0	0
	7	boy	2	0	0	0	0	2	0	1	1
		girl	0	2	1	0	0	3	1	0	2
	total	boy	2	0	0	0	0	2	0	0	1
		girl	0	2	2	0	1	3	2	1	2
		total	2	2	2	0	1	5	2	1	3

								
stage	age \ answer		red is more	blue is more	same	red is more	blue is more	same
I	3	boy	1	3	0	1	0	2
		girl	1	1	0	0	0	1
	4	boy	2	2	0	4	0	0
		girl	5	0	0	3	0	1
	5	boy	2	3	0	2	0	3
		girl	2	3	0	4	0	1
	6	boy	2	3	0	2	0	2
		girl	2	2	0	2	0	2
	7	boy	1	0	0	0	0	1
		girl	2	0	0	0	0	2
	total	boy	8	11	0	9	0	8
		girl	12	6	0	9	0	7
		total	20	17	0	18	0	15
II	3	boy	0	0	0	0	0	0
		girl	0	0	0	0	0	0
	4	boy	0	0	0	0	0	0
		girl	0	0	0	0	0	0
	5	boy	0	0	0	0	0	0
		girl	0	0	0	0	0	0
	6	boy	0	0	0	0	0	0
		girl	0	1	0	0	1	0
	7	boy	2	0	0	2	0	0
		girl	0	1	2	0	0	3
	total	boy	2	0	0	2	0	0
		girl	0	2	2	0	1	0
		total	2	2	2	2	1	3

The children of no answer are omitted from this table

And, concerning 40 children in stage I, there were 8 children who decided the amount of liquid only by height, 5 children who decided only by the number of containers, and no one decided only by width.

Four children of the above 5 were bewildered by height when the number of containers are equal. So we get a conclusion; height have an effect when the number of containers is small, and number itself have an effect when the number of containers is large.

〔Notice〕

Though there were 2 girls (3 years old) who answered "red is more", the reason is guessed they liked red colour. So, we need reexamine these 2 girls exchaging red colour for another one.

○ Stage Ⅱ

Toshiko (7 ; 9)

She verifies there is the same amount in the containers A_1 and A_2.

We pour A_1 into B_1 and B_2,

• "Which do you think you can drink muck more, $(B_1 + B_2)$ or A_2 ?"

"It is blue".

• "How was it before ?"

"It was the same"

• "But, now ?"

"The same."

• "Why ?"

"Because it was in the same container before."

We pour blue juice into C_1 and C_2.

• "How is it now ?"

"It is the same."

We pour a part of blue juice C_1 and C_2 into C_3.

• "How is it now ?"

"Red is more."

• "Why?"

"Because red is in large cup and this (blus) is in small cup."

We pour red juice (B_1 , B_2) into $C_4 \sim C_8$.

• "Which do yon think you can drink muck more, red juice or blue juice ?"

"It is red juice."

• "Why?"

"This (red) is more 2 cups."

• "In this time, I will pour this blue juice into this cup (A_2) . Watch it carefully."

We pour C_1, C_2 and C_3 into A_2.

• "Where will the red juice come up to when I pour it in this large cup (A_1) ?"

"About here (She points to the height as same as blue) ."

This girl has one of the characterization of stage Ⅱ which Piaget stated. That is to say, the child is capable of assuming that quantity of liquid will not change when it is poured from A into B_1 and B_2, but when three or more containers are used he falls back on to his earlier belief in non-conservation.〔5〕

Katsuyuki (7 ; 9)

He verifies $A_1 = A_2$. Then, we ponr A_1 (red) into B_1 and B_2.

• "Which do yon think you can drink much more, red juice or blue juice ?"

"Blue juice, no, red juice can be drunk much more."

• "Why ?"

"If I pour it back, I have got more, so I have more if I divide it".

- "How was it before ?"

"It was the same."

- "Now ?"

"Red juice is more."

- "Well, I also pour the blue juice into this two (C_1, C_2) cups. How about now ?"

"The same."

- "Why ?"

"If I pour B_1 into B_2, it becomes as high as C_1, but it is a half wide, so if I pour C_1 into C_2, both are the same. "

We pour a part of C_1 and C_2 into C_3.

- "Now, how is it ?"

"Blue."

- "Why ?"

"Because, at first those were the same two by two, but now blue juice is one more added."

We pour red juice (B_1, B_2) into $C_4 \sim C_8$.

- "Now, which do you think you can drink much more, this red juice or blue juice ?"

"......"

He is thinking silently by measuring of height and width using his finger.

- "Is it the same amount ?"

"No, red juice is more."

- "Why ?"

"Two red juices are more than one blue juice. The rest of them (it means $C_4 + C_5 - C_1$) and C_6 is equal to this (C_2). In that case, as I said before, two red juices are more than one blue juice, so red is much more."

- "Now I will pour blue juice into this cup (A_2). Watch it carefully. Well, I will pour all red juice into this cup (A_1), where will the red juice come up to ?"

"...... (He points to the level which is higher than that of blue juice)."

This child multiplies logically width, height and number of the cups, but he can not recognize the constancy of whole quantity. He explains clearly and every time he is asked, he explains the reason as before. Though he recognized B_1+B_2 equal C_1+C_2, this would be due to the facility of multiplicative relations. This child is talented in the field of language, and he has got the multiplication logique of relations, but the conservation of quantity is not achieved.

Kazumi (7 ; 9)

He verifies $A_1=A_2$.

- "I pour this red juice (A_1) into those two cups (B_1, B_2), which do you think you can drink much more, blue juice or red juice ?"

"Blue is more."

- "Why ?"

"The cup is large and has much more juice, those cups are small and have a little. "

• "They were the same before, but now one is more than the other, isn't it ?"

"Yes, much more."

We pour A_2 into C_1 and C_2.

• "How is it, now ?"

"I can drink the same amount."

• "Why ?"

"Because, I poured A_1 into B_1 and B_2, and I also poured A_2 into C_1 and C_2."

• "Then, I will subdivide blue juice into three cups. (We pour a part of C_1, C_2 into C_3). Now, which do you think you can drink much more ?"

"The same. "

• "Why ?"

"Because, you poured C_1 and C_2 into $C_1 \sim C_3$. "

We pour B_1 and B_2 into $C_4 \sim C_8$.

• "Now, how is it ?"

"The same. "

• "How do you understand it ?"

"Because, you poured $B_1 + B_2$ into $C_4 \sim C_3$."

We pour blue juice into A_2.

• "If I pour red juice into this cup (A_1), where will the red juice come up to ?"

"Here. (He points to the same level as the blue juice.)"

This child recognized conservation of liquid, only except between A_2 and $B_1 + B_2$. What is the reason, why ? When I asked him "They were the same before, but now one of them is more, isn't it ?", he did not recognize the conservation, but he answered to the later questions "It is the same, because it is only poured into the other containers. " What is the reason, why?

This is a novel instance which we can not find in Piaget's example. As the feature of stage Ⅱ, 5 children among 6 recognize conservation of $B_1 + B_2$ and $C_1 + C_2$. Therefore, it would be easy to recognize the conservation when the number of cups is equal.

○ Stage Ⅲ

Atsushi (5 ; 9)

(a) He verifies $A_1 = A_2$. After that, we pour A_1 into B_1 and B_2.

• "Which do you think you can drink much more, here (red juice $B_1 + B_2$) or there (blue juice) ?"

"Here (red juice) ."

• "Why ?"

"Because, blue is in this cup, but red is devided into two cups."

• "Both were the same before, weren't they ? But now ?"

"Oh, yes. (No answer after that) ."

• "Which do you think you can drink much more ?"

"Blue is more."

We pour A_2 into C_1 and C_2.

- "Which do you think you can drink much more, red juice or blue juice "

"Red."

We pour blue juice into $C_1 \sim C_3$.

- "Which do you think you can drink much more ?"

"Blue."

- "Look. this cup (red) is bigger."

"Red is more."

- "Look, this cup (blue) is high."

"Blue is more."

We pour red juice into 5 small cups $C_4 \sim C_8$.

- "Now ?"

"Red."

- "Why ?"

"It (red) has much more juice."

Blue juice is poured back into A_2.

- "If I pour all red juice into A_1, how far up will it come ?"

"Here (He points to the same height as blue"

- "Why ?"

"Because, before the juices were the same."

As this child gave the right answer to the experiment of multiplicative relations (iii) easily, we repeated the experiment (i) again, and the results were following.

(b) He recognizes $A_1 = A_2$. After that we pour A_1 into B_1 and B_2.

- "Which do you think you can drink much more, this red juice or that blue ?"

"It is the same."

- "Why ?"

"Because they were the same amount before."

After that, every time he was asked, thinking for a while he gave the right answer saying "They were the same before." Thus, he grasped the conservation of quantity.

Though this child seems to discover the constency of quantity directly without the multiplicative relations, it is due to the very fact that he has mastered the multiplicative relations.

Takashi (7 ; 9)

He verifies $A_1 = A_2$. Then we pour A_1 into B_1 and B_2.

- "Which do you think you can drink much more ? Are they the same ?"

"Yes, they are the same."

- "Why ?"

"Because, even if it is devided into two cups, the water does not decrease."

We pour A_2 into C_1 and C_2.

- "Now ?"

"It is the same."

- "How do you know ?"

"Because, it's only been poured out."

We pour a part of blue juice (C_1, C_2) into C_3.

- "Now ?"

"It's the same all the time."

And he always give right answer "It's the same."

This child is explaining as if it is natural "The water is not decrease even if it is divided into two." He is quite in stage Ⅲ, and though the children in this stage all answer calmly as if *a priori* they have percepetd conservation, they must have grasped multiplicative relations. In fact, these children all gave right answer to the experiment of multiplicative relations (iii) .

Now, though we elected 3 ～ 7 years old children as examinee, classifying it in 3 stages we get (Table 2) .

The chilcren who are 3 ～ 6 years old almost belong to stage Ⅰ. In this point of view, our results different from that of Piaget. That is, though he cites[6] many children of 5～ 6 years old as example of middle reaction (stage Ⅱ) , in our experiment the child who belongs to stage Ⅱ is only one.

(Tab. 2) Classification of exp. of partition (i)

age	sex \ stage	Ⅰ	Ⅱ	Ⅲ	total
3	boy	5	0	0	5
	girl	3	0	0	3
4	boy	5	0	0	5
	girl	5	0	0	5
5	boy	5	0	1	6
	girl	5	0	0	5
6	boy	5	0	0	5
	girl	4	1	0	5
7	boy	1	2	2	5
	girl	2	3	0	5
total		40	6	3	49

(ii) **The experiment of partition**

We make red juice (A_1 , height 30*mm*) 4 *mm* lower than blue juice (A_2 , height 34*mm*) . We divide red juice into many cases, and each time children are asked which is more or not.

○ Stage Ⅰ

Seiichi (5 ; 2)

He recognizes $A_1 < A_2$ and we pour red juice (A_1) into B_1 and B_2.

- "Which do you think you can drink much more, red juice ($B_1 + B_2$) or blue juice (A_2) ?"

"It is the same."

- "Why ?"

"It becomes the same amount."

We pour (B_1 + B_2) into (C_1 + C_2) .

- "Which do you think you can drink much more, red juice ($C_1 + C_2$) or blue juice (A_2) ? Or, are they the same ?"

"Red."

- "Why ?"

"Because the cups are filled."

We pour (C_1 + C_2) into C_1 ～ C_4.

- "Which do you think you can drink much more, red juice or blue juice (A_2) ? Or, are they the same ?"
"Red."
- "Why ?"
"Because red juice is divided into 4 cups."
- "If I pour red juice ($C_1 + \cdots + C_4$) into large cup (A_1), how far up will it come ?"
"Here (He points to the same level as A_2)."

○ Stage Ⅱ
Syoichi (6 ; 0)
He verifies $A_1 < A_2$. We pour A_1 into B_1 and B_2.
- "Which do you think you can drink much more, red juice ($B_1 + B_2$) or blue juice (A_2) ? Or, are they the same ?"
"Blue."
- "Why ?"
"Because blue juice was filled before."
We pour ($B_1 + B_2$) into ($C_1 + C_2$) .
- "Which do you think you can drink much more, red juice ($C_1 + C_2$) or blue juice (A_2) ? Or, are they the same ?"
"Red."
- "Why ?"
"……"
We pour ($C_1 + C_2$) into $C_1 \sim C_4$.
- "Which do you think you can drink much more, red juice ($C_1 + \cdots + C_4$) or blue juice (A_2) ? Or, are they the same ?"
"It is blue."
- "Why ?"
"Because blue was very much more before."
- "If I pour red juice ($C_1 + \cdots + C_4$) into large cup (A_1) , where will it come up to ?"
"…… (He points to lower level or the same level as A_2)."

○ Stage Ⅲ
Takashi (7 : 9)
He verifies$A_1 < A_2$. Then we pour A_1 into B_1 and B_2.
- "Which do you think you can drink much more, red juice ($B_1 + B_2$) or blue juice (A_2) ? Or, are they the same ?"
"Blue."
- "Why ?"
"If you divid the less one into two cups, it is not changed."
We pour ($B_1 + B_2$) into ($C_1 + C_2$) .
- "Which do you think you can drink much more, red juice ($C_1 + C_2$) or blue juice (A_2) ? Or, are they the same ?"

"It is blue."

· "Why ?"

"Because, though this (C_1 + C_2) is two , this (A_2) was more before."

We pour (C_1 + C_2) into $C_1 \sim C_4$.

· "Which do you think you can drink much more, red juice ($C_1 + \cdots + C_4$) or blue juice (A_2) ? Or, are they the same ?"

"It is blue."

· "Why ?"

"Because, this (A_2) was more before."

Classifying examinee in 3 stages, we get (Table 3) .

As for the children of stage Ⅰ, according to (Tab. 4) , some of them answer red (A_1) is more and some of them answr blue (A_2) is more, but when they are asked for the reason "Why? ", they answer only "Because they are divided into two cups", "I could not understand", "Because they are little" or "Because it is filled." But these answers belong to better field. Our great trouble is, when youngsters are asking for the reason, there are many who give no answer at all.

Let's classify the answer of children who belongs to stage Ⅰ. Then we get (Table 4).

(Tab. 3) Classification of exp. of partition (ii)

age	sex \ stage	Ⅰ	Ⅱ	Ⅲ	total
3	boy	5	0	0	5
	girl	3	0	0	3
4	boy	5	0	0	5
	girl	5	0	0	5
5	boy	5	0	1	6
	girl	5	0	0	5
6	boy	4	1	0	5
	girl	5	0	0	5
7	boy	1	1	3	5
	girl	0	2	3	5
total		38	4	7	49

(Tab. 4) Red juice (A_1) < Blue juice (A_2)

Answer to question "Which is more ? "

age \ answer	reb ($B_1 + B_2$)	the same	blue (A_2)	reb ($C_1 + C_2$)	the same	blue (A_2)	red ($C_1 + \cdots + C_4$)	the same	blue (A_2)
3	2	0	1	2	1	0	1	0	0
4	6	0	3	7	0	1	5	0	2
5	3	1	6	8	0	2	5	0	5
6	6	0	3	4	0	5	5	0	2
7	0	1	0	0	0	1	1	0	0

Omit the child who give no answer

Though the children of stage Ⅱ answer correctly to the 1 st question, they cannot answer to the 2nd and 3rd question. But even if their answers are wrong, their answers are different from that of the children who belong to st–

age I . For example, they are suffering from the recognization "Small cup is narrow in width and large cup is wide in width."

Those who in stage III confirm conservation not to disturbing by the form and the number of containers, and they reach to stability.

(iii) **The experiment of multiplicative relations**

There are the same amount of juice ; red in A_1 and blue in A_2. Pouring A_1 into L, we require to have the same amount as A_2.

○ Stage I

Yuji (4 ; 8)

He verifies $A_1 = A_2$.

- "Pour as much red juice into this one (L) as there is blue (A_2) ."

"...... (He pours all red juice) ."
- "Are they the same ?"

"No."
- "Is red more ?"

"Yes."
- "What must you do to have the same amount ?"

"Make the levels equal."
- "Is that right ?"

"Yes."

As for this child's reaction, in spite of he recognizes the equality of the contents of containers A_1 and A_2, pouring A_1 into L (long and narrow) he alreädy deny the equality of them. He does not recognize the reversibility of operaion at all, and he grasps the quantity merely perceptual data. But, nevertheless, he does not feel any contradiction. He thinks that the quantity of liquid also varied because of the containers have been changed.

Yoshiaki (3 ; 10)

He verifies $A_1 = A_2$.

- "Can you pour in this (L) as much as blue ?"

"Yes (He pours all red juice into L) ."
- "Are they the same ?"

"Yes."
- "Which is much more ?"

"Red."

What does this example, in the world, mean ? Can't the child understand the meaning of the problem ? I think these must be deeply study from the language point of view.

When children are asked "Pour as much from A_1 into L as there is in A_2," they pour all contents of A_1 into L. I think that they remark only to "pour into L" and they are not attentive to "as same as." In short, this question contains two factors; "same quantity" and "to pour into", but they can't think both two factors simultaneously.

○ Stage Ⅱ

Kazuhiko (4 ; 8)

He verifies $A_1 = A_2$.

· "Pour as much red juice into L as there is blue."

"Yes (He pours all red juice into L)."

· "Can you drink the same ?"

"Red is more."

He pours some red juice back operating by himself.

· "Now, how do you think ?"

"Still red juice is more."

He makes the height as same as blue juice.

· "Is that right ?"

"This time, blue becomes much more."

He adds red juice a litte.

· "Now, how do you think ?"

"Red is more."

He makes the levels as the same, and cries

"Blue is more."

In this secondary stage Ⅱ, these chidren are trying to co-ordinate the relations of height and width. We find here progress from stage Ⅰ. But the relations of hight and width are not so strong, that the level of one liquid becomes higher than the other, the multiplicative relations diminish and the perception prevails. And when the levels become same, tha relation of width appears, and they become aware of "Blue is more."

Hisako (6 ; 4)

She recognizes $A_1 = A_2$.

· "Pour as much red juice into this (L) as there is blue."

"…… (She makes the level as same as blue)."

· "Are they the same ?"

"Blue is more."

· "Make them the same."

"… (She pours red juice into L as high as 2 times of blue juice)."

· "Is it the same ?"

"Yes."

· "Is there the same amount to drink ?"

"Yes. I can drink the same."

The children at this stage, co-ordinate the multiplicative relations of height and width. But as the multiplicative relations are weak, they are satisfying when the level of L becomes a little higher than that of A_2. This is the reason which they can't be at stage Ⅲ. We can't find these kinds of children in Piaget's work. As Piaget starts from $A_1 \fallingdotseq A_2$, it would be imagine that if the children make the height of L a little higher than A_2, doesn't he belong them at stage Ⅲ ?

○ Stage Ⅲ

Atsushi (5 ; 9)

He virifies $A_1 = A_2$.

- "Pour as much red juice into this (L) as there is blue."

"… (He pours red juice into L till it is ½ full, and he is thinking about, then he makes the level as same as blue)."

- "Can you drink the same ?"

"Blue is more."

- "What must you do to have the same amount ?"

"… (At first he pours the liquid pointing to the level which is a little higher than that of blue, after that he pours the whole)."

- "Are they the same ?"

"Yes."

- "Why ?"

"Because they were the same before."

- "But this (L) is higher, isn't it ?"

"Well, but this is narrow."

Masanori (7 ; 6)

- "Pour as much red juice into this (L) as there is blue."

"… (He pours the whole)."

- "Can you drink the same ?"

"Yes."

- "L is higher, isn't it ?"

"But, at first they both were the same."

In the former, while he is handling, at last he grasps the reversibility of

(Tab. 5) The classification of the multiplicative relations (iii)

age	sex \ stage	Ⅰ	Ⅱ	Ⅲ	total
3	boy	4	1	0	5
	girl	2	1	0	3
4	boy	4	1	0	5
	girl	5	0	0	5
5	boy	2	3	1	6
	girl	4	1	0	5
6	boy	2	3	0	5
	girl	2	3	0	5
7	boy	0	0	5	5
	girl	0	1	4	5
total		25	14	10	49

operation. In the later, the constancy of quantity is established from begining.

We can get (Table 5) by classifying the examinee into 3 stages.

(iv) **The relations among three experiments**

Ⓐ As for the relations of the experiment of partition (i) and (ii) ,we notice that both of them are developing parallelly. The children who can do the experiment (i) can do the experiment (ii), and the children who can't do (i) can't do (ii). And whether the children have constancy of the liquid or not depend on whether they can think over "level of the liquid", "size of the containers" and "number of the containers" etc. simultaneously or not respectively.

The children who don't yet grasp conservation decide quantity of the liquid by one of these conditions; level of the liquid, size of the containers and number of the containers. As we showed it in some examples of the experiments, it is decided by a very instinctive and perceptive factor. That is to say the change of appearance leads to the change of quantity and the children don't suspect it. There is a long way to reach the conservation as they can't still regard as illusion as illusion.

On the other hand, the children who can recognize the conservation can connect it directly with constancy of the quantity without affecting by appearances. It is interesting that there are two ways of its appearence. One is due to the reversible operation, that is the children can reback to "the contents of the containers are the same before", and other is due to the fact that they can multiplicate the relations between height and width, that is they explain the reason "though it is higher but it is narrow." As for these two ways, there exist constancy of the liquid in background.

Ⓑ Secondly, concerning the relations between the experiment of multiplicative relations (iii) and the experiment of partition (i) & (ii) , we must notice that there are many children who are bad in (i) & (ii) , but are good in (iii) .

What's the reason ? There are many kinds of containers and they are used in (i) & (ii) . Therefore, as for "numben of the containers" the experiment (i) & (ii) are more complicated than that of (iii) , so in (i) & (ii) the children will be disturbed. In other words, to multiplicate the relations between "width" and "height" —— multipication logique —— is contained both experiments, but the children must multiplicate another "number of the containers" in experiment (i) & (ii) , so it would be accompanied difficulties in (i) & (ii) .

As it is mentioned above, cognition of partition and that of multiplicative relations don't develop parallelly, but cognition of multiplicative relations would rather be completed earlier than that of partition.

To see easily the mentioned above, let's put (Tab. 2), (Tab. 3), and (Tab. 5) into (Tab. 6) where each child is made clear in what stage he belongs.

(Tab. 6)

age	3				4				5				6				7		
exp. \ child	(i)	(ii)	(iii)	exp. \ child	(i)	(ii)	(iii)	exp. \ child	(i)	(ii)	(iii)	exp. \ child	(i)	(ii)	(iii)	exp. \ child	(i)	(ii)	(iii)
b_1^3	I	I	I	b_1^4	I	I	I	b_1^5	I	I	II	b_1^6	I	I	I	b_1^7	III	III	III
b_2^3	I	I	I	b_2^4	I	I	I	b_2^5	I	I	I	b_2^6	I	I	II	b_2^7	II	III	III
b_3^3	I	I	I	b_3^4	I	I	II	b_3^5	I	I	I	b_3^6	I	I	I	b_3^7	II	I	III
b_4^3	I	I	II	b_4^4	I	I	I	b_4^5	III	III	III	b_4^6	I	II	II	b_4^7	III	III	III
b_5^3	I	I	I	b_5^4	I	I	I	b_5^5	I	I	II	b_5^6	I	I	II	b_5^7	I	III	III
g_1^3	I	I	I	g_1^4	I	I	I	b_6^5	I	I	II	g_1^6	I	I	I	g_1^7	II	III	III
g_2^3	I	I	I	g_2^4	I	I	I	g_1^5	I	I	II	g_2^6	I	I	I	g_2^7	I	II	III
g_3^3	I	I	II	g_3^4	I	I	I	g_2^5	I	I	I	g_3^6	II	I	II	g_3^7	I	III	III
				g_4^4	I	I	I	g_3^5	I	I	I	g_4^6	I	I	II	g_4^7	II	II	III
				g_5^4	I	I	I	g_4^5	I	I	I	g_5^6	I	I	II	g_5^7	II	II	III
								g_5^5	I	I	I								

Where bi (i=3,4,···,7) means i years old boy
bj (j=1,2,···,6) means examinee boy
g is an abbreviation for girl

§5. Conclusions

Let's compaire the results of our experiment of partition (i) & (ii) and the experiment of multiplicative relations with that of Piaget.

As Piaget says, the experiment of partition (i) & (ii) is classified in 3 stages. In stage I, it seems to us that the children put their attention only one relation (for example level of the liquid), but they can't adjust the other relations (for example, width and number of the containers) with former one. In stage II, it's a period of transition and completion, and conservation gradually emerges. In stage III, they postulate conservation of the quantities immediately.

The children who are at stage III, will be founded in (Tab. 6) ; one child at 5 years old, two children at 7 years old, there are three children in all in the experiment (i) , and one child at 5 years old, six children at 7 years old, there are seven children in all in the experiment (ii). So, we can say that the experiment (ii) is easier than that of (i).

Secondly, let's show the different point from Piaget's results. As for the reaction of stage Ⅱ Piaget says ; the child is capable of assuming that the quantity of liquid will not change when it is poured from glass A into two glasses B_1 and B_2, but when three or more glasses are used he falls back on to his earlier belief in non-conservation.[7]

According to our results, we can lead the different result from Piaget. For example, it is Kazumi (stage Ⅱ, experiment (i) , § 4) .

That is to say, he confirms $A_1 = A_2$, then we pour A_1 into B_1 and B_2 and ask him "Which do you think you can drink much more, $B_1 + B_2$ or A_2 ?". His answer to this question is A_2 or $B_1 + B_2$ is much more. But after we pour A_2 into C_1 and C_2, ask him "Which do you think you can drink much more, $B_1 + B_2$ or $C_1 + C_2$?", he gives right answer "It is the same." And after that even if number of the containers is greater, he always gives right answer. We doubt, though this child has already reached at stage Ⅲ, the lack of fitness of our question let him stay at stage Ⅱ. But every time the child was confirmed, he repeated the same answer, we can't help accepting a first incorrect answer as it is.

Well, how shall we interpret this ? When we divided a cup of juice into two there was no conservation, and when we divided two cup of juices into two respectively he acknowledged conservation, so I want to interpret these facts that two cup of juices subdivid into two containers respectively conservation of the liquid suddenly flashs in him, therefore he maintains conservation continually for other successive questions.

The results of the experiment of multiplicative relations also agree with that of Piaget. According to our results the children who are at stage Ⅲ will be founded in (Tab. 6) ; one child at 5 years old, nine children at 7 years old, so there are ten children in all.

From this, we can conclude that experiment (iii) is the easiest, and from experiment (ii) to (i) difficulties are increased gradually.

Next, let's show the different point from Piaget's results. He has no intermediate reactions between stage Ⅱ and Ⅲ, but we can find it in our experiment. For example, it is Hisako (stage Ⅱ, experiment (iii) , § 4) .

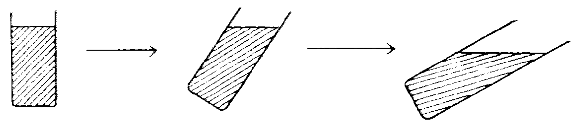
That is to say, at first she verifies $A_1 = A_2$. Then she pours A_1 into L until its level is as same as A_2 and she stops. But finding that A_2 is more, she pours A_1 into L until its level is two times of A_2, and she satisfies. She is on the verge of stage Ⅲ.

Though Piaget does not recognize such example, it would depend upon his poor way of the experiment as I explained at the place of Hisako. In short, as the way which Piaget adopted here was $A_1 \neq A_2$, it would be very difficult to ascertain whether she had poured the same amount into L or not.

Now, as we mentioned above, we could grasp the real situation on child's cognition of conservation of continuous quantities. We could find that if we instruct children from 7 years old, we shall get great effect.

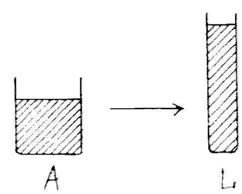
Well, what is the desirable course of study to introduce cognition of the contionuous quantities ? Let's end this treatise stating my own thought.

① Ore seizes the conservation of continuous quantities by using continuous deformation of one container.
(take one container, incline it, and deforming surface of the liquid gradually.)



② By using containers A, A′ (these are a little different in width or volume etc.) , one confirms the conservation by to pour A into A′ and by contrary to pour A′ into A.

③ One confirms the conservation by to pour A into L (which is long and fine container) and to give difference in height.



④ When $A_1 \neq A_2$ (to pour liquid of different volume into equal containers A) , one confirms the conservation by to pour A_1 into B_1 and B_2, and to pour A_2 into B_3 and B_4.

⑤ When $A_1 = A_2$, one confirms the conservation by the same method as ④.

⑥ Then, let's extend to the other case.

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Abstracted

On the conservation of continuous quantities, J. Piaget has detailed in his work "The Child's Conception of Number 1952 Routledge & Kegan Paul (p. p. 3~24)."

As this is, of course, the results for the children of his native place Switzerland, I determined to reexamine the Japanese children.

The results of the investigation are generally the same as his assert, that is to say the children are classified in three stages Ⅰ, Ⅱ & Ⅲ, but are not agree with in detail.

Now, supporsing that we could know about the conservation of continuous quantities of the Japanesen children, what is the desirable course of study to introduce the conservation of continuous quantities ? Our answer to this question will be stated in the following.

① One seizes the conservation of continuous quantities by using continuous deformation of one container. (take one container, incline it, and deforming surface of the liquid gradually.)

② By using containers A, A′ (these are a little different in width or volume ete.), one confirms the conservation by to pour A into A′ and by contrary to pour A′ into A.

③ One confirms the conservation by to pour A into L (which is long and fine container) and to give difference in height.

④ When $A_1 \neq A_2$ (to pour liquid of different volume into equal containers A), one confirms the conservation by to pour A_1 into B_1 and B_2, and to pour A_2 into B_3 and B_4.

⑤ When $A_1 = A_2$, one confirms the conservation by the same method as ④.

⑥ Then, let's extend to the other case.